Topic: Inverse hyperbolic derivatives

Question: Find the derivative of the inverse hyperbolic trig function.

$$y = \cosh^{-1} x^3$$

Answer choices:

A $y' = \dfrac{3x^2}{\sqrt{x^6 - 1}}$ with $x < 1$

B $y' = \dfrac{3x^2}{\sqrt{x^6 + 1}}$ with $x < 1$

C $y' = \dfrac{3x^2}{\sqrt{x^6 + 1}}$ with $x > 1$

D $y' = \dfrac{3x^2}{\sqrt{x^6 - 1}}$ with $x > 1$

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Solution: D

Make a substitution, letting $u = x^3$ and $u' = 3x^2$. Then $y = \cosh^{-1} u$, so

$$y' = \frac{1}{\sqrt{u^2 - 1}} \cdot u'$$

$$y' = \frac{1}{\sqrt{\left(x^3\right)^2 - 1}} \cdot 3x^2$$

$$y' = \frac{3x^2}{\sqrt{x^6 - 1}} \text{ with } x > 1$$

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Topic: Inverse hyperbolic derivatives

Question: Find the derivative of the inverse hyperbolic trig function.

$$y = \tanh^{-1}\left(2x^5 - 1\right)$$

Answer choices:

A $y' = \dfrac{5}{2x + 2x^6}$ with $x \neq 0, \pm 1$

B $y' = \dfrac{1}{2x - 2x^6}$ with $x \neq 0,1$

C $y' = \dfrac{5}{x - x^6}$ with $x \neq 0,1$

D $y' = \dfrac{5}{2x - 2x^6}$ with $x \neq 0,1$

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Solution: D

Make a substitution, letting $u = 2x^5 - 1$ and $u' = 10x^4$. Then $y = \tanh^{-1} u$, so

$$y' = \frac{1}{1 - u^2} \cdot u'$$

$$y' = \frac{1}{1 - \left(2x^5 - 1\right)^2} \cdot 10x^4$$

$$y' = \frac{10x^4}{1 - \left(4x^{10} - 4x^5 + 1\right)}$$

$$y' = \frac{10x^4}{1 - 4x^{10} + 4x^5 - 1}$$

$$y' = \frac{10x^4}{4x^5 - 4x^{10}}$$

$$y' = \frac{5}{2x - 2x^6} \text{ with } x \neq 0,1$$

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Topic: Inverse hyperbolic derivatives

Question: Differentiate the function.

$$f(x) = \left(\sinh^{-1}\left(2x^3\right)\right)^4$$

Answer choices:

A $$f'(x) = \frac{24x^2\left(\sinh^{-1}\left(2x^3\right)\right)^3}{\sqrt{x^2+1}}$$

B $$f'(x) = \frac{24x^2\left(\sinh^{-1}\left(2x^3\right)\right)^3}{\sqrt{4x^6+1}}$$

C $$f'(x) = \frac{6x^2\left(\sinh^{-1}\left(2x^3\right)\right)^3}{\sqrt{x^2+1}}$$

D $$f'(x) = \frac{6x^2\left(\sinh^{-1}\left(2x^3\right)\right)^3}{\sqrt{4x^6+1}}$$

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Solution: B

To find the derivative of the given function

$$f(x) = \left(\sinh^{-1}\left(2x^3\right)\right)^4$$

we'll apply chain rule, taking the derivative of the outermost function first, while we leave the inner function untouched. We'll work our way from the outside to the inside. The outermost function is a power function, so we'll apply power rule.

$$f'(x) = 4\left(\sinh^{-1}\left(2x^3\right)\right)^3 \frac{d}{dx}\left(\sinh^{-1}\left(2x^3\right)\right)$$

The derivative of $\sinh^{-1}$ is given by

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$$

so we get

$$f'(x) = 4\left(\sinh^{-1}\left(2x^3\right)\right)^3 \left(\frac{1}{\sqrt{(2x^3)^2+1}}\right) \frac{d}{dx}\left(2x^3\right)$$

Take the derivative of the last, innermost power function, then simplify.

$$f'(x) = 4\left(\sinh^{-1}\left(2x^3\right)\right)^3 \left(\frac{1}{\sqrt{(2x^3)^2+1}}\right)\left(6x^2\right)$$

$$f'(x) = 4\left(\frac{6x^2}{\sqrt{4x^6+1}}\right)\left(\sinh^{-1}\left(2x^3\right)\right)^3$$

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$$f'(x) = \frac{24x^2\left(\sinh^{-1}\left(2x^3\right)\right)^3}{\sqrt{4x^6+1}}$$

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